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FORMULATION OF THE SINGULAR INTEGRAL EQUATION TECHNIQUE FOR GENERAL PLANAR TRANSMISSION LINES

Abbas S. Omar and Klaus Schünemann

Technische Universität Hamburg-Harburg, Arbeitsbereich
Hochfrequenztechnik, Postfach 90 14 03, D-2100 Hamburg 90, W-Germany

ABSTRACT

The singular integral equation technique is used to determine the normal modes of propagation in general planar transmission lines. Taking fin-lines as example it is demonstrated, how high-order modes can effectively and accurately be calculated. Some of these modes show an unusual evanescent nature for certain combinations of parameters. The modes of this type exist always in pairs with their squared propagation constants being complex conjugate.

INTRODUCTION

General planar transmission lines are defined here as multi-layer structures with an arbitrary number of air-dielectric and/or dielectric-dielectric interfaces where conducting strips are mounted. The accurate determination of the normal modes of propagation in these guiding structures is of fundamental importance. Due to the completeness property of the set of normal modes /1/, an arbitrary electro-magnetic field inside the guiding structure can be expanded within this set, so that the problem of determining the field, which can usually be formulated as a solution of integro-differential equations, is reduced to a solution of matrix equations. This includes, for example, the problems of scattering by conducting or dielectric obstacles inside guiding systems /2/, scattering by apertures in the boundaries /3/, and discontinuities between different guiding systems /4/. Galerkin's method in the spectral domain had been successfully used for eigenmode calculations by almost all authors, e.g. in /5/. As has been shown in /6/, this method is superior with respect to computational efforts if the dominant and the first few higher-order modes are to be analyzed. However, if modes of still higher order are to be determined, this method becomes computationally time consuming. The problem of analyzing these high-order modes is as yet unsolved.

The singular integral equation technique characterizes the problem by a characteristic matrix of relatively low order even in the case of high-order modes. Hence it will be applied here to the analysis of general planar transmission lines with the generalized unilateral fin-line structure in Fig. 1 as illustrating example. In particular, it will be shown that a matrix of order 7 is quite sufficient to yield accurate results up to the 30th mode. Another inherent advantage is that all matrix elements are given analytically so that neither infinite sums nor numerical integrations are involved.

BASIC FORMULATION

The electromagnetic field in the generalized unilateral fin-line structure of Fig. 1 is a linear combination of LSM and LSE fields /1/.

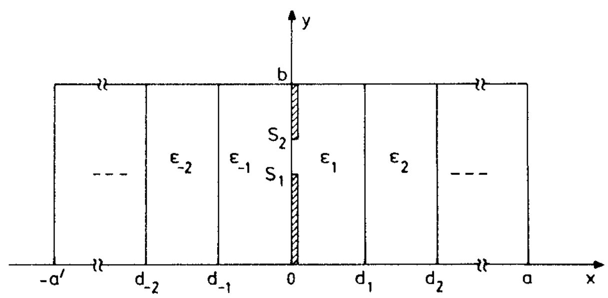

Fig. 1: Generalized unilateral fin-line

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These both parts independently satisfy all interface conditions. They are, however, coupled in order to fulfill the edge condition on the conducting fins /6/. Hence we can write for the tangential electric field $\underline{E}_t$ and for the surface current $\underline{J}_s$ at the interface x = 0:

$$\underline{E}_t = \underline{E}_t^e + \underline{E}_t^h , \ \underline{J}_s = \underline{J}_s^e + \underline{J}_s^h$$

with superscripts "e" and "h" referring to LSM and LSE parts, respectively.

It has been shown in /6/ that the LSM part is completely characterized by the z-components E_z^e and J_z^e, because:

$$E_y^e \sim dE_z^e/dy \ , \ J_y^e \sim dJ_z^e/dy$$

whereas the LSE part is completely characterized by the y-components E_y^h and J_y^h, because:

$$E_z^h \sim dE_y^h/dy \ , \ J_z^h \sim dJ_y^h/dy$$

Taking the perfectly conducting walls at y = 0 and y = b into account, the tangential electric field and surface current components can be written as:

$$E_z^e = \sum_{n=1}^{\infty} A_n^e \sin(n\pi y/b),$$

$$E_y^h = \sum_{n=0}^{\infty} A_n^h \cos(n\pi y/b),$$

$$J_z^e = j\omega\varepsilon_0 \sum_{n=1}^{\infty} F_n^e A_n^e \sin(n\pi y/b),$$

$$J_y^h = (1/j\omega\mu_0) \sum_{n=0}^{\infty} F_n^h A_n^h \cos(n\pi y/b) \qquad \ldots(1)$$

F_n^e and F_n^h can be shown to represent the Fourier series expansion coefficients of the LSM and LSE Green's functions, respectively, /7/.

The singular integral equation technique can now be formulated by the following steps:

1. Two cosine-series $f_1(y)$ and $f_2(y)$ are constructed in terms of the tangential electric field $\underline{E}_t$ so that they are only unknown in the slot region ($s_1 \leq y \leq s_2$):

$$f_1(y) = \sum_{n=0}^{\infty} A_n^{(1)} \cos(n\pi y/b),$$

$$f_2(y) = \sum_{n=0}^{\infty} A_n^{(2)} \cos(n\pi y/b) \qquad \ldots(2)$$

   The expansion coefficients $A_n^{(1)}$ and $A_n^{(2)}$ are linear combinations of A_n^e and A_n^h.

2. Two sine-series $f_3(y)$ and $f_4(y)$ are constructed in terms of the surface current $\underline{J}_s$ in a way that their coefficients asymptotically approach $A_n^{(1)}$ and $A_n^{(2)}$, respectively, so that the expansion coefficients of the series $g_1(y)$ and $g_2(y)$, defined as:

$$g_1(y) = \sum_{n=1}^{\infty} A_n^{(1)} \sin(n\pi y/b) - f_3(y),$$

$$g_2(y) = \sum_{n=1}^{\infty} A_n^{(2)} \sin(n\pi y/b) - f_4(y) \qquad \ldots(3)$$

   are vanishing asymptotically.

3. Applying the boundary conditions to be satisfied by $f_1(y)$ and $f_2(y)$, the coefficients $A_n^{(1)}$ and $A_n^{(2)}$ are determined in terms of integrals of $f_1(y)$ and $f_2(y)$, respectively, which are taken over the slot region.

4. Substituting these integrals into equations (3) and applying the boundary conditions to be satisfied by $f_3(y)$ and $f_4(y)$, one arrives at two integral equations relating $f_1(y)$ and $f_2(y)$ to $g_1(y)$ and $g_2(y)$, respectively. These integral equations are of standard singular type. Their solutions are well-known (see e.g. /8/).

5. Because their expansion coefficients are asymptotically vanishing, $g_1(y)$ and $g_2(y)$ may be truncated at the N-th term. Hence $f_1(y)$ and $f_2(y)$ are then known in the slot region in terms of $A_n^{(1)}$ and $A_n^{(2)}$, ($n \leqq N$).

6. A back-substitution of $f_1(y)$ and $f_2(y)$ in the integrals determining $A_n^{(1)}$ and $A_n^{(2)}$, ($n \leqq N$) -step 3-, in conjunction with additional conditions (which will be discussed later) results in a finite, homogeneous system of linear equations, from which the propagation constants and the field expansion coefficients of the different modes can be calculated.

In applying the singular integral equation technique to the general planar structure, one must be careful in constructing $f_1(y)$, $f_2(y)$ and $f_3(y)$, $f_4(y)$, which are related to $\underline{E}_t$ and $\underline{J}_s$, respectively. Both E_y, J_z and dE_z/dy, dJ_y/dy are singular at the fin edges, ($y = s_1$, $y = s_2$) /9/. The order of singularity is $|y - s|^{-1/2}$ for all cases. This is the proper singularity, the singular integral equation technique can deal with. Hence any differentiation of either E_y, J_z, dE_z/dy or dJ_y/dy with respect to y is not allowed in the construction of $f_1(y)$, $f_2(y)$, $f_3(y)$ and $f_4(y)$.

There are two ways of how to construct these functions. The first is to express $A_n^{(1)}$ and $A_n^{(2)}$ as linear combinations of A_n^e and A_n^h, so that the LSM and LSE parts of the field are coupled from the first beginning. This will be called the coupling formulation. The second way it to choose $A_n^{(1)} \sim A_n^e$ and $A_n^{(2)} \sim A_n^h$, so that the LSM and LSE parts are decoupled. The coupling will then be taken into account as a final step, as will be shown below. This will be called the decoupling formulation.

THE COUPLING FORMULATION

Functions $f_1(y)$ and $f_2(y)$ are constructed as:

$$f_1(y) = dE_z/dy,$$

$$f_2(y) = \left[(k_o^2-\beta^2)E_y+j\beta(dE_z/dy)\right]/j\omega\mu_o \qquad \ldots(4)$$

and $f_3(y)$ and $f_4(y)$ as:

$$f_3(y) = \left[(K^h\beta^2-K^e k_o^2)J_z+j\beta K^h(dJ_y/dy)\right]/j\omega\varepsilon_o K^e K^h,$$

$$f_4(y) = \left[j\beta(K^e-K^h)J_z+K^h(dJ_y/dy)\right]/K^e K^h \qquad \ldots(5)$$

K^e and K^h are the asymptotic limits of $(n\pi/b)F_n^e$ and $F_n^h/(n\pi/b)$, respectively.

It can be seen that the vanishing of $f_1(y)$ and $f_2(y)$ on the fins and of $f_3(y)$ and $f_4(y)$ in the slot region (the boundary conditions to be considered) are just necessary conditions for the vanishing of $\underline{E}_t$ and $\underline{J}_s$ on their respective regions (i.e. $\underline{E}_t$ on the fins and $\underline{J}_s$ in the slot). Thus E_y and J_z will vanish on their respective regions, E_z and J_y, however, can still assume a non-zero constant value on their respective regions. Because points y = 0, y = b (where E_z = 0) belong to the fins, this constant is automatically zero as far as E_z is concerned. Hence it is sufficient to impose one additional condition on J_y, namely its vanishing at any point within the slot region.

THE DECOUPLING FORMULATION

We now construct the various functions according to:

$$df_1/dy = \underline{\nabla}_t \cdot \underline{E}_t = -\Delta_t E_z^e/j\beta,$$

$$f_2(y) = \hat{\underline{i}}\cdot(\underline{\nabla}_t \times \underline{E}_t) = \Delta_t E_y^h/j\beta \qquad \ldots(6)$$

$$f_3(y) = -\underline{\nabla}_t \cdot \underline{J}_s/j\omega\varepsilon_o K^e = -\underline{\Delta}_t J_z^e/\omega\varepsilon_o\beta K^e,$$

$$df_4/dy = j\omega\mu_o\left[\hat{\underline{i}}\cdot(\underline{\nabla}_t \times \underline{J}_s)\right]/K^h = \omega\mu_o\Delta_t J_y^h/\beta K^h \qquad \ldots(7)$$

$\underline{\nabla}_t$ and Δ_t mean del- and Laplacian operator in y-z plane, respectively, and $\hat{\underline{i}}$ is the unit vector in x -direction.

The boundary conditions to be imposed on the four functions (which are: $df_1/dy = 0 = f_2(y)$ on the fins and $f_3(y) = 0 = df_4/dy$ in the slot) are again necessary ones for the vanishing of $\underline{E}_t$ and $\underline{J}_s$ on their respective regions. They guarantee only that the individual components of $\underline{E}_t$ and $\underline{J}_s$ are harmonic functions on their respective regions so that additional conditions have again to be imposed. They can be shown to just be the vanishing of one component of $\underline{E}_t$ and of $\underline{J}_s$ on the boundaries of their respective regions, i.e. at $y = s_1$, $y = s_2$.

ACCURACY FOR HIGH-ORDER MODES

There is only one approximation involved into the singular integral equation technique, namely the truncation of the infinite series $g_1(y)$ and $g_2(y)$ at the N-th term. It can, however, be proved that the n-th coefficients of these series are negligible if $(n\pi/b)^2 > (\varepsilon_{r-max}k_o^2-\beta^2)$. ε_{r-max} means the maximum ε_r in the multi-layer structure. This inequality actually puts an upper limit on the number of high - order modes which can be accurately calculated, because the propagation constant of the highest-order modes must satisfy:

$$|\beta|^2 < \left[(N+1)\pi/b\right]^2-\varepsilon_{r-max}k_o^2$$

For dimensions and dielectric constants normally used, N = 3 (corresponding to a characteristic matrix of order 7) is sufficient to give accurate results for the first 30 modes.

FIN-LINE MODES WITH COMPLEX β^2

For certain combinations of parameters (i.e. frequency, slot width, ... etc.), it has been found that β^2 of one or more pairs of evanescent modes are no longer real. Instead, they have been found as complex conjugate pairs in the complex plane.

Let β^2 of one of these pairs be β_1^2 and β_2^2. The square roots which are physically possible for a z-dependence $e^{-j\beta z}$ and a time dependence $e^{j\omega t}$ can be written as

$$\beta_1 = \beta - j\alpha \ , \ \beta_2 = -\beta - j\alpha$$

where β and α are positive. This means that one mode propagates in the +z-direction and is attenuated in the same direction. The other mode is also attenuated in this direction, it propagates, however, in the -z-direction, i.e. it represents an "attenuated backward wave". From the first view, it is easily stated that a mode with complex propagation constant β_1 propagates in the same direction, in which it is attenuated. This means a continous energy loss, although the structure has been assumed lossless. The other mode with propagation constant β_2 propagates to the opposite direction in which it is damped. This means a continous energy gain, although the structure is passive. This point of view would be correct only if the two modes were not coupled. In fact, it has been found that the two modes are so strongly coupled, that the electric field of each mode does not couple to its own magnetic field, but to the magnetic field of the other mode, i.e.

$$\int_S (\underline{e}_1 \times \underline{h}_1^*)\cdot\underline{ds} = 0 = \int_S (\underline{e}_2 \times \underline{h}_2^*)\cdot\underline{ds} \qquad \ldots(8)$$

$$\int_S (\underline{e}_1 \times \underline{h}_2^*)\cdot\underline{ds} = p \neq 0, \int_S (\underline{e}_2 \times \underline{h}_1^*)\cdot\underline{ds} = -p^* \qquad \ldots(9)$$

Here $\underline{e}_1(\underline{h}_1)$ and $\underline{e}_2(\underline{h}_2)$ are the transverse electric (magnetic) field vectors of the modes with propagation constants β_1 and β_2, respectively, and S is the fin-line cross-section. Eqs. (8), (9) mean that each mode cannot exist alone: both should always exist together, if they exist.

Now we investigate these modes from the energy point of view. Let us assume that the two questionable modes are excited (by e.g. a certain discontinuity). Because each of these modes it not coupled to the other modes which may also be excited, it is sufficient to study the energy contained in these two modes only. Let $\underline{E}_t$ and $\underline{H}_t$ be the transverse electric and magnetic field vectors, respectively, of the two superposed modes, i.e.

$$\underline{E}_t = A_1 e^{-j\beta_1 z}\underline{e}_1 + A_2 e^{-j\beta_2 z}\underline{e}_2, \underline{H}_t = A_1 e^{-j\beta_1 z}\underline{h}_1 + A_2 e^{-j\beta_2 z}\underline{h}_2$$

Integrating the pointing vector over the fin-line cross-section, and making use of eqs. (8) and (9), one obtains

$$P = \int_S (\underline{E}_t \times \underline{H}_t^*)\cdot\underline{ds} = j\ W \sin(2\beta z + \varphi)\ e^{-2\alpha z}$$

where $A_1 A_2^* p = -(W/2)\ e^{-j\varphi}$. The vanishing of the real part of P guarantees that the two superposed modes carry no power, i.e. they behave as a whole evanescently. The energy stored in these superposed modes oscillates along the line once being inductive and once being capacitive in nature with a superposed exponential decay.

Finally, we would like to state that, although this phenomenon has been found in fin-lines, we believe that it also exists in all other planar guiding structures, at least in those with closed conducting boundaries.

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